TAILORING SYMBOLIC CLOTHES FOR ANCIENT LOGIC

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Abstract

In order to assess the applicability of modern formal systems to ancient Indian logic, it is worth looking into the development of European logic, where, earlier than in indology, tools of modern formal logic have been used for the research into ancient logical systems. Frege's quantifier logic, invented during the last quarter of the 19th century, not only led to a revolution regarding the utilization of formal logic in the area of the foundations of mathematics, but also with regard to the use of formal logical methods as a tool for the interpretation of ancient logical texts. Whereas the triumphal march of quantifier logic in the field of mathematics for which it had been designed was doubtlessly justified, the situation in the area of history and philosophy of logic encountered obstacles. Not until FREGE's interpretation of Aristotelian syllogistics, which was subsequently copied into almost every elementary logic textbook, had been overthrown by the eminent modern formal logician Jan Łukasiewicz, did the new ill-fitting Fregean clothes of ancient logic fall off, and the field for a new debate on the formal aspects of Aristotle's logic become open again. This led in 1972 to John Corcoran's now widely accepted formal interpretation of Aristotelian syllogistics. Starting with Łukasiewicz' student Stanisław Schayer around 1930, research in the field of ancient Indian logic also switched tools from Aristotelian syllogistic to modern symbolical logic. As in the case of ancient European logic, there is a risk that philosophically insensitive misapplication of this method distorts ancient texts and injects artificially generated problems, thus creating obscurity and confusion instead of clarity and precision.

Introduction

Up to the middle of the last century, Aristotelian syllogistics was *the* standard instrument or, at least, provided the vocabulary for eminent scholars of ancient Indian logic. In his "Storia della filosofia indiana" published in 1957, TUCCI (1992) continuously makes use of the Aristotelian mode of speaking of *minor*, *middle*, and *major terms* denoted by S, M, and P, respectively. He writes $S\grave{e}P$ (S is P)[1], and he apparently takes for granted some acquaintance of classical syllogistic reasoning on the part of the reader.

In the passage "Il sillogismo"[2] he formulated the five-membered syllogism of the Nyāya-Vaiśeṣika school as follows:[3]

[1] but also "*tutto* $S\grave{e}P$"

[2] Tucci (1992), p. 197 ff.

[3] My translation.

Thesis	- there is fire on the mountain	$S\grave{e}P$
Reason	- because there is smoke	$S\grave{e}M$
Example	- where there is smoke, there is fire, like in the kitchen	$M\grave{e}P$
Application	- there is smoke on the mountain	$S\grave{e}M$
Conclusion	- there is fire on the mountain	$S\grave{e}P$

I will not step into the details or even the problematic issues of Tucci's interpretation. I just want to point to the fact that he uses the Aristotelian way of presentation quite naturally. Many other authors before Tucci had also chosen Aristotelian terminology for the presentation of their results, three of the most prominent of them being Vidyābhūṣaṇa (1920)[4], Stcherbatsky (1962) and Randle 1924[5].

Modern formal logic arrived in the field of indology in 1932, when the Polish indologist Stanisław Schayer published two papers[6] on the formal interpretation of Nyāya logic, based on the modern calculus of predicate logic. Due to the influence of Jan Łukasiewicz,[7] Schayer,

> ... obviously following Łukasiewicz' successful application of formal logic to the study of the ancient Greek logicians, ... did the same with regard to ancient Indian logic. [8]

Schayer's attitude towards the use of formal logic in indology is the following:[9]

> We do not have a satisfactory account of Indian syllogistics ... [which has] to do with the fact that those indologists who have written about formal-logical aspects of Nyāya so far are not familiar with European logic.
>
> ... it is time to approach the Nyāya texts with logical, not just with philosophical or philological competence.
>
> If Athalye, Vidyābhūṣana, and Stcherbatsky thought that the Indian syllogism can be traced back to the Barbara mode [of Aristotelian logic] then this is a misunderstanding which ought to disappear from indological literature.

We will now present Schayer's "formal clothes" to the so-called "five-membered Indian syllogism" (Schayer (1933))[10]:

[4]In Appendix B of Vidyābhūṣaṇa (1920), "Influence of Aristotle on the development of the syllogism in Indian logic", the author writes (p. 499): "In order, therefore, to ascertain whether there is any genetical connextion between the syllogism of Indian Logic and that of Greek Logic, I shall analyse here the two syllogisms side by side, with occasional references to the rules controlling them." And, after 12 pages of "side-by-side" comparison, he comes to the following conclusion (p. 511): "Considering the antiquity of the syllogism as propounded by Aristotle and the close connexion that exists between it and the syllogism promulgated in the Hindu Logic, we may fairly conclude that the latter was greatly influenced by, if not based on, the former. ... From the stages in the development of the syllogism in Hindu Logic, as indicated above, it will appear that Aristotle's works migrated into India during three distinct periods."

[5]In Chapter III, "Syllogism (Parāthānumānam)", Stcherbatsky compares the European with the Indian syllogism, and he arrives at the conclusion (p. 317): "There is a great difference between the European and the Buddhist syllogistic theory. ... The solution proposed by Dignāga and Dharmakīrti is, in some respects, nearer to Kant and Sigwart than to Aristotle."

[6]The original German texts, Schayer (1932) and Schayer (1933), were reprinted in Schayer (1988). We will quote the English translation given in Ganeri (2001).

[7]It is not known whether there was a personal connection between Schayer and Łukasiewicz, who, for some years, were both teaching at Warsaw University, Poland. Most probably, Schayer attended Łukasiewicz' introductory lectures delivered at Warsaw University in 1928/29. For more details, cf. Glashoff (2004).

[8]Mejor (2003)

[9]Schayer (1933)

[10]We use a slightly different notation for predicate logic than that employed by Schayer.

1) *pratijñā*	$\psi(a)$	There is fire on a (= on this mountain)
2) *hetu*	$\varphi(a)$	There is smoke on a (= on this mountain).
3) Statement of *vyāpti*	$\forall x\varphi(\text{x})\rightarrow\psi(\text{x})$	For every locus x: if there is smoke in x then there is fire in x
4) *upanaya* = statement of the *pakṣadharmatā*	$\varphi(a) \rightarrow \psi(a)$	This rule also applies to $x= a$ (for the pakṣa)
5) *nigamana* = statement of *sādhya*	$\psi(a)$	Because the rule applies to x = a and the statement φ(a) is true, the statement ψ(a) is true

Again, we will not step into the really problematic details of this formalization (for a detailed critique see Glashoff (2004)).

The utilization of Aristotelian logic in Indology faded out during the decades after Schayer's work and modern symbolic logic became the almost exclusive instrument. A typical example is the "rehabilitation of the five-membered syllogism" by Oetke (1994b).

Today many different systems of formal logic have been developed, almost generating a situation of excess supply of formalisms. This leads to a kind of uncritical overconsumption on the side of those (few) indologists who are equipped with a certain armamentarium of logical tools. The use of formal systems is backed by the prevalent opinion that modern symbolic logic is a universal and neutral instrument, enabling an impartial and complete analysis of any situation in which it can be applied. In order to show that this is simply not true, and that a straightforward utilization of symbolic logic for the interpretation of ancient texts may do more harm than good, it pays off to have a closer look at what happened in European logic after philosophers switched over from traditional Aristotelian term logic to Fregian quantor logic.[11] We will show that it took around 100 years to straighten out the problems introduced by a naive application of the modern quantorial formalism to the interpretation of Aristotle's logic. This should give ample food for thought to indologists who plan to enrich their scientific papers by a stylish assortment of formulas.

The classical subjects of Aristotelian logic

In this section we present a very short introduction into the main terminology of Aristotle's logic, which is a prerequisite for understanding the different ways of putting this logic into modern clothes.

Classical Aristotelian assertoric syllogistics of the *Analytica Priora* is divided into three main subjects:

- The doctrine of categorical terms
- The doctrine of categorical propositions
- The doctrine of categorical syllogisms .

Categorical terms

Aristotle denoted categorical terms (short: *terms*; Greek: *horoi*; Latin: *termini*) by the capital Greek letters $A, B, \Gamma, \Delta, ...$, and he gave many examples for what terms stand for:

[11] Mulhern (1974) describes some of the problems which arose in the fields of prearistotelian, Aristotelian, Stoic, and Commentatorial logic when modern symbolic logic was used for the interpretations of ancient texts.

- living being, man, animal, horse, swan, raven etc.
- good(ness), ignorance, wisdom, wild(ness), inanimateness etc.
- substance, gold, metal, snow,
- science, number, line, state
- white(ness), black(ness)

We will denote categorical terms by $x, y, z, \ldots$ etc. There is an ongoing dispute on the question whether terms stand for abstract concepts or for sets of individuals. Aristotle's text allows both interpretations at different passages, and the abovementioned examples show that there are cases (like "living being" etc.) where the "extensional" interpretation (where terms denote sets of individuals) suggests itself more than an "intensional" one, where terms are considered to stand for general concepts (like "wisdom" etc.) or mass terms like water, metal, snow etc.. At the zenith of traditional logic in the 17th and 18 th century, the intensional interpretation prevailed (Arnauld (1861), Jungius (1957), Kant (1991)), but already LEIBNIZ had a precise understanding of either possibilities which he considered as being on an equal footing (see Kauppi (1960)).

Speaking in modern terms, the difference between extensional and intensional interpretations lies completely in the domain of *semantics*, and this can be totally uncoupled from the *syntax* of Aristotelian logic.[12] This difference has nevertheless influence on the way one would construct a symbolic language in which terms are the main building blocks: If the "intended semantics" of terms are general concepts and mass terms, then there is no need for introducing a special logical equipment for dealing with individuals. If however the formal system has to be designed to cover sets of distinct individuals, variables standing for individuals will be indispensable.

We will go into some details of this problem in the next section on categorial propositions.

Categorical propositions

Categorical propositions (short: propositions; Greek: *protasis*; Latin: *propositio*) are a particular kind of sentences, combining two different terms x and y. There are four different types of categorical propositions (see Cohen (2008); Boger (1998); Smith (2009)):

Type	Tag	Idiomatic	Technical	Abbr.
Universal Affirmative	A	Every x is a y	y belongs to every x y is predicated of every x	Ayx
Universal Negative	E	No x is a y	y belongs to no x y is predicated of no x	Eyx
Particular Affirmative	I	Some x is a y	y belongs to some y y is predicated of some x	Iyx
Particular Negative	O	Some x is not a y	y does not b. to some x y belongs to not every x	Oyx

While it was Aristotle who invented the technique of using "variables" representing terms, and while the abbreviation A, E, I, O had been in use since mediaeval times, it was Leibniz who, in the framework of his project of a *calculus universalis*, utilized formulas for sentences like Ayx etc. for the first time in the history of logic.

[12] An intensional "Leibnizian" semantics of Aristotelian logic is given in Glashoff (2010).

Let us return to the question of whether individuals should be included into the formalism. The categorial sentence

metal is predicated of *gold*

or, equivalently,

gold is *metal*

containing the general concepts *gold* and *metal,* could be simply formulated as

$$Amg,$$

as it is usually done in traditional logic. The relation between concepts which denote sets of individuals, for example, between *animal* and *horse*, can of course be written in the same way:

Aah (*animal* is predicated of *horse*),

but here we may also say

All horses are animals, i.e.

Every (individual) *horse* is an *animal*,

which then transforms into something like

Every*thing* which is a *horse* is an *animal*,

where now the "everything" contains a placeholder, the *index* "thing" for an abstract individual which, in this case, may be the bearer of the *properties* horse and animal. The "every" of "everything" may be regarded as a quantor. Exactly this construction is Frege's invaluable contribution to the development of modern logic and, specifically, modern mathematics. However, this construction is not always appropriate, as indexing with respect to individuals does not make much sense in case of mass terms like "water" and abstract singular terms like "black(ness)" etc. In the case of concepts describing sets of discrete individuals (horse, animals, etc.), both alternatives make sense.

Categorical syllogisms

Syllogisms are rules which allow to deduce, starting from a set of given propositions (the premises), additional propositions called conclusions. If, for example, the following premises are given:

- All penguins are birds
- No birds are mammals

then the rules of the Aristotelian system (in this instance, the rule is the syllogism called *Celarent*) generate, as conclusion, the new proposition

- No penguins are mammals

Aristotle has stated 14 different rules of this type which he called *sullogismoi.* He considered two of these rules (*Barbara* and *Celarent*) as basic and derived the remaining ones by a formal method called "reduction". (see Corcoran (1974a), Boger (1998)).

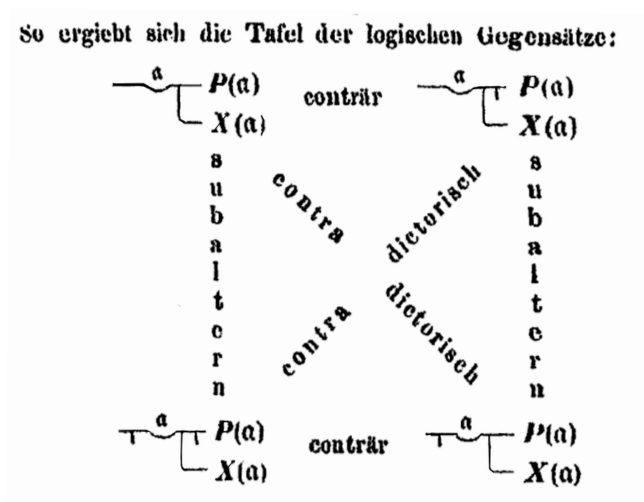
Frege's formalization by predicate logic

The "classical" formalization by predicate logic

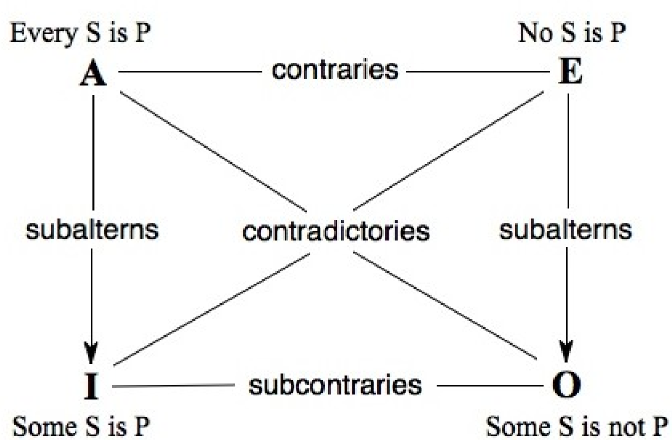
In his "Begriffschrift" of 1879[13], Frege proposed the following translation of the four Aristotelian judgements into his newly developed quantifier-predicate logic (here as well as in the following text we make use of the modern standard notation):

$$
\begin{array}{lllll}
Ayx & := & \forall\xi(x(\xi) \rightarrow y(\xi)) & & \\
Eyx & := & \forall\xi(x(\xi) \rightarrow \neg y(\xi)) & = & \neg\exists\xi(x(\xi) \wedge y(\xi)) \\
Iyx & := & \exists\xi(x(\xi) \wedge y(\xi)) & & \\
Oyx & := & \exists\xi(x(\xi) \wedge \neg y(\xi)) & = & \neg\forall\xi(x(\xi) \rightarrow y(\xi))
\end{array}
$$

He then presents the following table which is his version of the Square of Opposition ("Tafel der logischen Gegensätze") of Aristotelian logic:



which looks like an equivalent of the traditional Square of Opposition (Parsons (2004)):



[13]Frege (1967)

Why it doesn't work (technically)

However, it is easy to realize that Frege's diagram does not completely cover the situation of the classical diagram: While, according to the Prior Analytics, I is *subaltern* to A, i.e.,

$$Ayx \text{ implies } Iyx,$$

this is not true for the predicate-logical interpretation of Frege, as

$$\forall\xi(x(\xi) \to y(\xi)) \to \exists\xi(x(\xi) \wedge y(\xi))$$

is *not* a theorem of predicate logic! This implies that Aristotle's theory of syllogism, in which subalternation plays a vital role, *cannot* be reproduced by Frege's formalism. In addition, E does not entail O, furthermore A and E are no longer contraries, and I and O are not subcontraries. It is not astonishing that some of the syllogisms[14] of the Aristotelian theory do not hold true.[15]

Most modern logicians reacted to the obvious unsuitability of modelling Aristotelian syllogistics by Frege's method by putting the blame for this fact completely on Aristotle and his successors. It became fashionable to claim that in Aristotelian logic there was always the silent presupposition of "nonempty terms".[16] This theory of "existential import" then found its way easily into the formalization of Aristotelian logic, and here began its long not yet finished journey leading to hundreds, maybe thousands of articles and to many modern books parrotting this subject not because it was a real problem of Aristotelian logic but solely a special feature of the method of research into it.

We will take up the story of "existential import" later in a separate section.

Why it is philosophically questionable

After many scholarly quarrels about a correct formalization of Aristotelian syllogistics, the following remarks of Novak, published 1980 in the *Notre Dame Journal of Formal Logic*,[17] reflect the now widely accepted view of most scholars in the field of Aristotelian logic:

> Statements which today are represented with either the universal or existential quantifier were expressed by Aristotle without them. ... There is no special term in Aristotle for the concept of quantifier; such an expression occurs only later in Theophrastos ...
>
> Aristotle's most used phrase for expressing what in traditional logic is known as "All A is B", is "B belongs (*gr. hyparchei*) to all A" ... the employment of quantifiers necessitates the use of indexials; quantification is made over a "dummy subject" rather than the predicates itself. Thus, whereas Aristotle would write "Animal is predicted of man" (An. pr. 25a25), and thereby employ only general names in his syllogistic, the representation of the first order calculus will also imploy the indexial $\forall x(Mx \to Ax)$, where M stands for 'man' and A for 'animal'. ... Aristotle makes no clear distinction between abstract singular terms and concrete general terms and there is no positive evidence to indicate that he ever tried to reduce statements containing either kind of terms to statements making reference only to individuals.
>
> ... the question arises what might be the referent of the indexial $'x'$? Aristotle's own presentation leads one to believe that he is dealing with relations between what is

14

For example, AAI in the first figure, known as *Barbari*.

15 See Strawson (1952), pp. 167-169.

16 This expression does not make sense. A term cannot be said to be empty or nonempty, because it is purely syntactic device. By talking about "emptiness of a term", one obviously refers to a semantic concept which however is hardly ever mentioned.

17 Novak (1980), p. 231-232

directly represented by the general terms whereas in the functional representation the original subject and predicate terms of an Aristotelian proposition only indirectly fall under reference. A realistic construal of the terms, i.e. "For all things such that they possess the property man, they also possess the property animal," is not wholly appropriate to Aristotle's approach. Such a translation seems to speek of "something" being a substance of this or that sort, of possessing this or that property.

One must reproach modern interpretations of ancient Indian logic, from Schayer (1932) to several recent authors for committing exactly the same fault of misinterpreting or even twisting texts of Indian logic in order to force them into the bed of quantifier logic. We will give a detailed example in the last section of this paper.

A digression on existential import

Strawson's first amendment

As a remedy to the "faults" of Aristotelian logic, modern logicians tried to introduce a symbolic variant of "existential import" into the quantifier-logic definition by augmenting Frege's definition of A and E by additional terms. Strawson (1952)[18] writes:

> ... by adding an existential commitment to the universal forms [A and E], this interpretation may seem to possess the additional advantage of bringing us a little closer to the most frequent uses of 'all' and 'no' forms in ordinary speech.

This leads to the following set of formulas:

$$\begin{array}{lcll} Ayx & := & \forall\xi(x(\xi) \to y(\xi)) & \wedge \quad \exists\xi(x(\xi)) \\ Eyx & := & \neg\exists\xi(x(\xi) \wedge y(\xi)) & \wedge \quad \exists\xi(x(\xi)) \\ Iyx & := & \exists\xi(x(\xi) \wedge y(\xi)) & \\ Oyx & := & \exists\xi(x(\xi) \wedge \neg y(\xi)) & \end{array}$$

While this looks reasonably at first sight , grave consequences arise, as Strawson immediately pointed out:

> It is true that this second interpretation saves the validity of some of the traditional laws which have to be sacrificed on the first [Fregian interpretation]; but not all of them. And this limited success is gained at the cost of rendering yet other laws of the old system invalid.

Specifically these definitions fail to save the rule that I and O cannot both be false, and it does not preserve the simple conversion of E. So Strawson comes to the conclusion:

> Either the A and E forms have existential import or they do not. If they do, one set of laws has to be sacrificed as invalid; if they do not, another set has to go. Therefore no consistent interpretation of the system as a whole, within the prescribed limits, is possible.

[18] p. 169

Strawson's second amendment

Strawson then performs a "kind of ad hoc patching up of the old system in order to represent it, in its entirety, as a fragment of the new."[19] After performing a set of "manœvers" he arrives at a final interpretation which reads as follows:

$$
\begin{array}{lllllll}
Ayx & := & \forall\xi(x(\xi)\to y(\xi)) & \wedge & \exists\xi(x(\xi)) & \wedge & \exists\xi(\neg y(\xi)) \\
Eyx & := & \neg\exists\xi(x(\xi)\wedge y(\xi)) & \wedge & \exists\xi(x(\xi)) & \wedge & \exists\xi(y(\xi)) \\
Iyx & := & \exists\xi(x(\xi)\wedge y(\xi)) & \vee & \neg\exists\xi(x(\xi)) & \vee & \neg\exists\xi(y(\xi)) \\
Oyx & := & \exists\xi(x(\xi)\wedge \neg y(\xi)) & \vee & \neg\exists\xi(x(\xi)) & \vee & \neg\exists\xi(\neg y(\xi))
\end{array}
$$

He concludes that it is not possible to put "existential import" into a satisfying symbolic form:

> For this interpretation, all the laws of the traditional logic hold good together; and they hold good within the logic of classes or quantified formulae; as a part of that logic.
>
> So the consistency of the system can be secured in this way. But the price paid for consistency will seem a high one, if we are at all anxious that the constants 'all', 'some', and 'no' of the system should faithfully reflect the typical logical behaviour of these words in ordinary speech.[20]

Jaskowski's negative result

The three possibilities of formalization presented above all use the monadic predicate calculus.[21] One may ask whether there are, within this calculus, "better" formalizations of Aristotelian logic. This question has been completely answered in the negative by Stanisław Jaskowski, the famous Polish mathematician of the Lvov-Warsaw School, a student of Jan Łukasiewicz, in his paper Jaskowski (1969). He showed: There is no formalization of Aristotelian logic within the framework of monadic predicate logic which

1. saves the validity of all traditional rules[22], and
2. has any connection to the colloquial meaning of the logical constants 'all', 'some' etc.

Jaskowski's paper[23] was the tombstone to the idea of interpreting Aristotelian syllogistics by tinkering with Frege's formulas. Let us at the end of this digression cite the words of Nedzynski (1979):

> The problem of existential import developed along with the development of modern symbolic logic during the nineteenth century. The problem is peculiar to the standard predicate calculus. There never was a real problem of existential import within the traditional syllogistic logic - it was placed there in retrospect by the modern logicians.

[19]Strawson (1952), p.171

[20]p. 173

[21]i.e. a calculus with one-place predicates only.

[22]In addition to what is contained in the Square of Opposition this includes identity as well as the rule of obversion.

[23]As there is a finite number (2^{16}different cases) of possible formalization of such a structure, I could confirm Jaskowski's result by a computer program in Glashoff (2006)

The road to a satisfying formalization

Łukasiewicz' formalization of Aristotle's syllogistics

In his famous book, first published in 1951[24], the Polish logician Łukasiewicz gave a new interpretation of Aristotle's syllogistics. Like many modern logicians he held that Aristotle relied on an "underlying" logic. Łukasiewicz main theses were:

- Aristotle used but never explicitely articulated the basic rules of propositional calculus as underlying logic
- his syllogistics was an axiomatic theory in which syllogisms are axioms or theorems.

In modern notation, Łukasiewicz' system has as sentences expressions of the form Axy and Ixy, where x, y are term-variables, and A, I are logical constants. E and O are given in terms of A and I by negation. In addition to the axioms of propositional logic, as well as to the rules of substitution and detachment, the axioms of the systems are

Axx	*A-Identity*
Ixx	*I-Identity*
$Ayz \wedge Axy \rightarrow Axz$	*Barbara*
$Ayz \wedge Ixy \rightarrow Ixz$	*Datisi*

The decisive difference of this translation in contrast to Frege's is that here there are no indexials: The variables now are the terms $x, y, ..$, not an indexial ξ referring to individuals.

We have seen that from a philosophical/philological standpoint Frege's translation is questionable and from a formal standpoint it is simply wrong. Łukasiewicz' formalization is much better in either aspect. It was in fact the first serious application of a modern symbolic system to an ancient logic.

There is another formalization in the language of predicate logic in the spirit of Łukasiewicz, given by Mates (1965). Mates made use of predicate logic as underlying logic, like Frege, but his quantors are indexed by terms instead of individuals. Of all translations of Aristotle's syllogistics into quantorial predicate logic, Mates' is the most serious one. Nevertheless, it shares all the disadvantages of Łukasiewicz' interpretation regarding the role of axioms in Aristoteles' logic.

Frege	Łukasiewicz	Mates
$\forall\xi(x(\xi) \rightarrow y(\xi)) \rightarrow \exists\xi(x(\xi) \wedge y(\xi))$	$Axy \rightarrow Ixy$	$\forall x \forall y(Ayx \rightarrow Ixy)$
not provable	axiom	axiom
in predicate logic	of propositional logic	of predicate logic

In spite of all its merits, Łukasiewicz' interpretation drew fire from the side of philosophy. Łukasiewicz himself had already admitted that his use of "Datisi" in the axiom system had no philological background, but that it had been chosen by him out of formal reasons. In addition, Aristotle had never made use of the "identity axioms". But the most serious attack on that system came from another side.[25] A precise formulation of this critique has recently been given by John Corcoran as follows:[26]

> As incredible as this may seem today, before the 1970s, the dominant view was that Aristotle's system was not really an underlying logic: it was thought to be an axiomatic

[24] Łukasiewicz (1957)
[25] Austin (1952); Corcoran (1974b)
[26] Corcoran (2009)

theory that presupposed an underlying logic never articulated by Aristotle. This would call into question the view that Aristotle was the founder of logic (cf. Smith 1989; Corcoran 1994). How could Aristotle be the founder of logic if he never presented a system of logic?

An intermezzo in Hamburg

When the German logician Paul Lorenzen gave a lecture at the philosophical department of Hamburg University in Germany in the beginning of the 1960's, invited by Carl Friedrich v. Weizsäcker, one of the students of Weizsäcker had the spontaneous inspiration that Lorenzen's logical system, the "Operative Calculus" might also be suitable for a formalization of Aristotle's syllogistic.[27] This student, Kurt Ebbinghaus, who had taken a diploma in electrical engineering before studying philosophy in Hamburg published his dissertation[28] in 1964. He gave an interpretation of Aristotelian syllogistics as a calculus consisting of rules, not of axioms. This is close to the traditional way of presenting the syllogistics, discarded vehemently by Łukasiewicz, and it avoids building on any underlying logic. What Ebbinghaus in fact showed was how Aristotle *constructed* a formal logic.

Ebbinghaus' rules are as follows:

$$\begin{array}{lrcll} (R_1) & AeB & \rightarrow & BeA & \textit{e-Conversion} \\ (R_2) & AaB, BaC & \rightarrow & AaC & \textit{Barbara} \\ (R_3) & AeB, BaC & \rightarrow & AeC & \textit{Celarent} \end{array}$$

plus additional rules governing negation and indirect reasoning, and plus two other syllogisms of the first figure.[29] In this formalism, " $\rightarrow$ " does not denote material implication of propositional calculus but stands for the application of a rule to a sequence of signs. In the context of Lorenzen's operative calculus, "$\rightarrow$" and "," are "protological" constants which presuppose no logical laws but only the ability of a human beeing or a machine to apply transformation rules to a string of signs.

Ebbinghaus succeeded in translating Aristotle's "reduction" of the valid syllogisms to those of the first figure by using Lorenzen's concept of "admissable" rules[30].

Ebbinghaus showed that he had constructed an isomorphism to Aristotelian syllogistics by means of his rule system, and he also proved the isomorphism of his system to Łukasiewicz'. He himself mentioned one important missing item in his model of Aristotelian logic: His research is an operation completely situated within the syntactic domain and it does not include any consideration of semantics or ontology. This excludes the modelling of some important aspects of classical logic like *ecthesis*,[31] proof by counterexamples, and moreover it cannot give a *justification* of the basic rules of his system.

There is no doubt that Ebbinghaus' dissertation which remained almost unnoticed [32], was a big step into the right direction of modelling Aristotle's work as a construction of a logic underlying scientific reasoning.

[27] Ebbinghaus (2010)

[28] Ebbinghaus (1964)

[29] Corresponding to Aristotle's results (and to Ebbinghaus' too) all syllogisms can be "reduced" to the two syllogisms listed above.

[30] According to Lorenzen (1960) these are rules which can be added to the original rules without enlarging the set of derivable theorems.

[31] Later Smith (1983) constructed a system built upon formalizing *ecthesis* syntactically.

[32] I know of only one not very enthusiastic review (Hamlyn (1966)) of Ebbinghaus work (see Corcoran and Glashoff (2010)).

Corcoran's system

After Łukasiewicz' ground-breaking book, it was the work of John Corcoran (1972)[33] which opened a complete new era of research into Aristotle's logic. Corcoran not only discarded Łukasiewicz' two main theses, but he also specified a Tarski-style semantics. In addition, he proved a completeness theorem for his syntax-semantics pair. This was much more than that what had been achieved by Ebbinghaus and, moreover, Corcoran presented his finding in a different and much more known and established formal system namely, that of natural deduction.

Corcoran's system consists of the following rules (no axioms!):

Conversion Laws	Laws of Perfect Syllogisms
$Exy \vdash Eyx$	$Azy, Axz \vdash Axy$
$Axy \vdash Iyx$	$Ezy, Axz \vdash Exy$
$Ixy \vdash Iyx$	$Azy, Ixz \vdash Ixy$
	$Ezy, Ixz \vdash Oxy$

plus negation rules and a formal definition of indirect reasoning. Corcoran showed how, using these rules, Aristotle's way of deduction as well of reduction can be modelled by means of a calculus of natural deduction. It is neither necessary to assume that Aristotle had made use of an underlying logic like propositional calculus or even quantorial predicate logic, nor to state syllogisms as axioms and provable theorems like Łukasiewicz did.

Without going into the details of the system, let us just give an example of how close Corcoran's formalism is to Aristotle's text.[34]

Deduction of the syllogism *Camestres* (second figure)

Aristotle's text	Modern notation	
1. If x belongs to every y	1. Axy	
2. but to no z,	2. Exz	
? then neither will y belong to any z	? Eyz	To prove.
3. For if x belongs to no z,	3. Exz	2 repetition
4. then neither does z belong to any x;	4. Ezx	3 e-conversion
5. but x belonged to every y;	5. Axy	1 repetition
6. therefore, z will belong to no y,	6. Ezy	4,5 Celarent
7. neither will y belong to any z.	7. Eyz	6 e-conversion

Notes on Indian logic

As I am not an indologist, and as my knowledge of Sanskrit is as meager as the knowledge of many indologists in the field of formal logic, I will not try to translate ancient Indian texts into modern symbolic logic. I will instead take a passage translated by a renowned indologist[35]and then show why it seems to me that some of the mistakes committed during the last 100 years in Aristotelian logic have duplicated in the field of Indian logic.[36]

[33] About the same time, Smiley (1973) independently published his paper on syllogistics. He presented a similar formal system, but his method is very different from Corcoran's.

[34] Corcoran (1974a,c); Boger (2004); Smith (2009)

[35] Oetke (1994a)

[36] Even if it may sound so, this is not an extremely harsh critique as the indologists who made these mistakes are in good company with such renowned modern logicians like Frege and Russell.

In the Vādavidhi, ascribed to the Buddhist logician Vasubandhu of the fourth century CE, there is a definition of the concept of "inseparable connection" of a logical reason with a *probandum*:[37]

1. "The (logical) reason is the pronouncement of a property which does not occur without a such."
2. "Inference is the observation of an object not occurring without [the probandum] for someone who knows that."[38]

The author goes on by translating a passage of the text which contains a further explanation of the first definition given above:

> A thing which never occurs when such a [thing], i.e. [a thing] of the same kind as the thing which has to be proven, as. e.g. the noneternity of sound, does not exist is a property which does not exist without a such, as e.e. [the property of] origination from effort [does not occur without] non-eternity and smoke [does not occur without] fire.

Thus we are now given two concrete examples of *hetu* and *sādhya,* which are referred to by the translator as *"things":*

hetu	*sādhya*
origination from effort	non-eternity
smoke	fire

The author now continues:

> If we assume that the idioms in question are meant to state relations existing between properties it is also possible to regard the phrases as referring to the (numerical) identy of the property-bearer. In this case the expression 'A occurs if/when B exists' should be explicated as: 'A and B are properties of one and the same thing' or: 'There is a property-bearer x such that both A and B are properties of x.'

No doubt: It is 'possible' to replace relations between properties by relations of the property bearers, however this is not mentioned in the text. The terminus "thing", previously taken as a label for the properties h and s is now used for the ominous x, which is not to be found in the ancient text and which is introduced by the author with the obvious intention to steer safely into the haven of predicate logic. This reminds us of Frege's translation of Aristotles logic, and we are not surprised, that, after some pages, we find the two equivalent formulas

$$\neg(\exists x)(Hx \wedge \neg Sx)$$

and

$$\forall x(Hx \rightarrow Sx)$$

[37]This corresponds to the relation of a valid sign (middle term) to the major term in Aristotelian logic.

[38]A straightforward term logical formalization of this sentences would lead to

$$\neg Ix\neg y$$

which, by the rules of term logic, is equivalent to

$$Ayx.$$

Thus, the criterion given in the Vādavidhi is, in term logic, equivalent to what could be described as "pervasion": y pervades x ("All x are y").

as symbolization of “inseparable connection”. In order to perform a classification of possiblities of symbolizing this concept of inseparable connection, the author not surprisingly even employs the undead concept of “existential import”.

This starting point of a research project into an ancient Indian text does not promise much, and indeed, the whole highly sophisticated exposition of 144 pages never recovers from the mislead interpretation right from the start. Thus, if we have reason to suspect that the symbol “x” as well as the different versions of existential import are nothing else than problematic imports from predicate logic having no textual basis, then why should we let us drag into the details of a such a classification of possibilities, making use of these instruments? And why should any indologist who has most probably had no training in symbolic logic, concern himself with a monstrous[39] classification of interpretations like the following?[40]

$$
\begin{array}{ll}
ES_{-eva}1) & (\exists x)(Hx \wedge Sx) \\
ES_{-eva}2) & (\exists x)(x \neq p \wedge (Hx \wedge Sx)) \\
ES_{+eva}1) & (\forall x)(Hx \rightarrow Sx) \\
ES_{+eva}2) & (\forall x)((x \neq p \wedge Hx) \rightarrow Sx) \\
ES_{+eva}3) & (\forall x)(Hx \rightarrow Sx) \wedge (\exists x)(Hx \wedge Sx) \\
ES_{+eva}4) & (\forall x)((x \neq p \wedge Hx) \rightarrow Sx) \wedge (\exists x)(x \neq p \wedge (Hx \wedge Sx))
\end{array}
$$

Let me close with a remark of Klima (2004), made in a similar context:

> When we engage a historical author by simply applying our own modern concepts in interpreting his claims, rather than trying to acquire his concepts, then there is always the serious danger of misinterpreting the author, who was thinking in a radically different conceptual framework.

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[39]Let me remark that these formulas are in fact syntactically correct. Other parts of the paper in question have added to my belief that most works on Indian logic in which of symbolic logic is utilized usually contain wrong, mishandled or “dadaistic” formulas. I cannot resist citing a particular blatant example for the latter which appeared in the Journal of Indian philosphy in 1975 written by another author :

	Dṛṣṭa-Warrant		$Hetu_{1-2} - Data$	*Conclusion*		*Implicit Assumption*
$(x)(iy)$	$(((Sx \supset Ix)$	.	$(Sy))\ \underset{\supset}{B}$	(Iy)	.	$(y \in x))$

This formula of course requires an additional legend much more extensive then the formula itself. The question arises why one should consider such formulas if they do not contribute to clarity and precision in the presentation of an ancient text!

[40]Oetke (1994a), p.24

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